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PHILOSOPHICAL MEMOIRS.

PART I.

ASTRONOMICAL AND MATHEMATICAL PAPERS.

I. *A Method of finding the Altitude and Longitude of the nonageſimal Degree of the Ecliptic; with an Appendix, containing Calculations from correſponding Aſtronomical Obſervations, for determining the Difference of Meridians between Harvard-Hall, in the Univerſity of* Cambridge, *in the Commonwealth of* Maſſachuſetts, *and the Royal Obſervatories at* Greenwich *and* Paris. *In a Letter from the Reverend* JOSEPH WILLARD, *Preſident of the Univerſity, and Correſponding Secretary of the American Academy of Arts and Sciences, to the Honorable* JAMES BOWDOIN, *L. L. D. Preſident of the Academy.*

SIR,

AS the parallaxes of the planets in latitude and longitude come frequently into uſe, in aſtronomical calculations, and particularly in deducing the difference of meridians between one place and another, from correſponding obſervations of ſolar eclipſes and occultations of fixed ſtars by the moon, every method which can be diſcovered, to ſhorten the work, or make it more eaſy, muſt be of utility. In calculating theſe parallaxes, it is neceſſary to find the altitude and longitude of the nonageſimal degree of the ecliptic. As I have given ſome attention to this ſubject, I take the liberty, Sir, of encloſing to you a paper, containing a method of finding theſe prerequiſites, different from any that I have happened to meet with, and (to me indeed) taking the whole proceſs together, eaſier, if not ſhorter. It is deduced from a projection which muſt make the method very obvious, to thoſe acquainted with the ſphere and with ſpheric trigonometry. You will find an appendix upon the longitude of Cambridge. If you think proper to communicate the whole to the Academy, you have my conſent.

I am,

with the greateſt reſpect, &c.

Cambridge, Auguſt 4, 1783.

DEFINITIONS.

1. THE right aſcenſion of the mid-heaven is that point of the equator, which culminates, or is in the meridian of any particular place, at a given time, and is found by adding the given apparent time, reckoned aſtronomically, to the ſun's right aſcenſion, calculated for the ſame time. Or, it may be found, by reducing the given apparent time to mean time, and adding it to the ſun's mean longitude. But, if the hours are reckoned according to civil time, we muſt take the difference between

the apparent given time and noon ; and if it is the forenoon, this difference muſt be ſubſtracted from the ſun's right aſcenſion ; but if it is the afternoon, it muſt be added. If the mean longitude is taken, the mean time muſt be uſed.

2. The nonageſimal degree of the ecliptic is its higheſt point above the horizon, and is 90° diſtant from each point of the ecliptic, where interſected by the horizon. This nonagiſimal point is determined by a perpendicular to the ecliptic, which paſſes through its poles and the poles of the horizon, the altitude of which point is meaſured from the horizon, upon this perpendicular.

PROBLEM.

Given the latitude of a place, the obliquity of the ecliptic, the time of the day, the ſun's true longitude, his right aſcenſion, and conſequently the right aſcenſion of the mid-heaven, to find the longitude and altitude of the nonageſimal degree of the ecliptic.

A GENERAL SOLUTION OF THE FOREGOING PROBLEM.

In plate I. fig. I. let the circle ♋p♑L repreſent the ſolſtitial colures ; then, the poles of the equator and of the ecliptic will be in the plane of this circle. Let the diameter E♎Q repreſent a portion of the equator ; ♋♎♑ a portion of the ecliptic ; P, the north pole of the equator ; p, the north pole of the ecliptic ; S and L their ſouth poles. Let the arc Pc S mark the right aſcenſion of the mid-heaven, from the neareſt equinoctial point ; then this arc will repreſent the meridian of ſome place, at a given time, and c and w will be the culminating points of the equator and ecliptic. Let the arc *bZi* repreſent the latitude of ſome given place; then the point Z, where the arc is interſected by the meridian, is the zenith of the place, at the given time, and is one of the poles of the horizon; the arc HOR is the horizon; the line ZN its axis, and the point N of the axis, where it is interſected by the arc Pc S continued, is the other pole of the horizon, or the nadir. The arc *p*ZL, drawn through the poles of the ecliptic and of the horizon, marks out the longitude of the nonageſimal degree upon the ecliptic at U, the altitude of which, or height above the horizon is VU. Now the arc *p*ZL is perpendicular both to the horizon and the ecliptic, as it paſſes through the poles of each, and as *p*U is = ZV = 90° and ZU is common to both, take ZU from each, and there will remain *p*Z = VU the altitude of the nonageſimal degree of the ecliptic. The longitude and altitude of this point, therefore, may be readily found by the ſpheric triangle P*p*Z, in which are given two ſides and the included angle, viz. ſide PZ = the co-latitude of the given place; the ſide P*p* = the diſtance of the poles of the equator and ecliptic = the obliquity of the ecliptic; and the angle *p*PZ, the value of which may always be determined by the following general rules, which, from diagrams, will appear very obvious.

The right aſcenſion of the mid-heaven being between 270° and 360° or 0°, and between 0° and 90°, the neareſt equinoctial point is ♈; and in the firſt caſe, angle *p*PZ is acute, and is found by ſubſtracting 270° from the right aſcenſion of the mid-heaven. In the ſecond caſe, the angle is obtuſe, and is found by ſubſtracting 270° from the right aſcenſion of the mid-heaven added to 360°, or, which is the ſame thing, by adding the right-aſcenſion of the mid-heaven to 90°.

The right aſcenſion of the mid-heaven being between 90° and 180°, and between 180° and 270°, the neareſt equinoctial point is ≏; and in the firſt caſe, angle pPZ is obtuſe, in the ſecond acute, and in each caſe the angle is found, by ſubſtracting the right-aſcenſion of the mid-heaven from 270°. Theſe rules are to be uſed when the latitude of the given place is north; but when it is ſouth, they muſt be inverted to find the correſponding angles, as will appear evident by a diagram.

From Z, let fall the perpendicular Zx upon the primitive circle: And to know whether the perpendicular, in any caſe, will fall upon the ſide of P next to p, or oppoſite to it, the following rules may be obſerved.

When the right aſcenſion of the mid-heaven is between 180° and 270°, or between 270° and 0°, that is, when angle pPZ is acute, the perpendicular will fall upon the ſide of P next to p; but when the right aſcenſion of the mid-heaven is between 0° and 90°, or between 90° and 180°, that is, when angle pPZ is obtuſe, the perpendicular will fall upon the ſide of P oppoſite to p.

For the ſide pZ, the altitude of the nonageſimal degree. Radius : Sine co-angle pPZ :: Tangent ſide PZ : Tangent ſide Px. Then, if the perpendicular be on the ſide of P next to p, the difference between Pp and Px will be px; but if it fall on the ſide of P oppoſite to p, the ſum of Pp and Px

will be px. px being found ſay, Sine co-Px : Sine co-px :: Sine co-PZ : Sine co-pZ.

For angle PpZ, or ∠ at the pole of the ecliptic, Sine ſegment px : Sine ſide Px :: Tangent angle pPZ : Tangent angle PpZ.

When the perpendicular Zx falls upon the ſide of P oppoſite to p, or between P and p, the angle PpZ is acute; but when it falls beyond p, it is obtuſe.

By the angle PpZ, the longitude of the nonageſimal degree is determined thus:—When the neareſt equinoctial point is ♈, ſubſtract this angle from ♋ or 3^s, adding 12^s to 3^s, when neceſſary; the remainder will be the longitude of the nonageſimal degree. If the neareſt equinoctial point is ♎, add this angle to ♋ or 3^s, and the ſum will be the longitude of the nonageſimal degree.

Let the foregoing general rules be now elucidated by a particular

EXAMPLE.

Required the longitude and altitude of the nonageſimal degree of the ecliptic, at the royal obſervatory at Greenwich, Auguſt 5, 1766, at $5^h\ 29'\ 57''$ P. M. apparent time?

The ſolar and lunar elements from MAYER's tables.

At $5^h\ 29'\ 57''$ P. M.

The ſun's longitude	$4^s\ 13^\circ\ 9'\ 51''$
The obliquity of the ecliptic	$23\ 28\ 18$

The

The latitude of Greenwich	51° 28′ 39″
Reduced to the center, the earth being an oblate ſpheroid	51 14 11
Complement = PZ	38 45 49

For the ſun's right aſcenſion.

Radius				10
: Sine of the co-obliquity of the ecliptic,	66°	31′	42″	9 9624911
:: Tang: ☉'s longitude from neareſt equinoct point,	46	50	9	10 0278498
: Tang: ☉'s right aſcenſion from neareſt point, viz. ♎	44	21	46	9 9903409
Subſtract the above from	180	0	0	
The remainder is ☉'s right aſc. from ♈	135	38	14	

The given time $5^h\ 29'\ 57''$ being reduced to degrees, at the rate of 15° per hour, gives 82° 29′ 15″. Add this to the ſun's right aſcenſion 135° 38′ 14″; the ſum 218° 7′ 29″ is the right aſcenſion of the mid-heaven. This ſubſtracted from 270° according to the rule, leaves 51° 52′ 31″ for angle pPz.

For ſide pZ, the altitude of the nonageſimal degree.

Radius					
: Sine Co ∠pPZ		38°	7′	29″	9 7905493
:: Tangent	PZ	38	45	49	9 9047023
: Tangent	Px	26	22	10	9 6952516
Subſtract	Pp	23	28	18	
Remains	px	2	53	52	

Sine

Sine Co	Px	63°	37′	50″	9	9522832
: Sine Co	px	87	6	8	9	9994443
:: Sine Co	PZ	51	14	11	9	8919475
: Sine Co	pZ	60	21	48	9	9391086
Alt. nona. deg.	pZ	29	38	12		

For angle Ppz

Sine	px	2°	53′	52″	8	7037573
: Sine	Px	26	22	10	9	6475369
:: Tangent ∠pPZ		51	52	31	10	1052425
: Tangent ∠PpZ		84	53	49	11	0490221
Obtuſe =		95	6	11		

To ♋ =	3s	0°	0′	0″
Add ∠PpZ =	3	5	6	11
The ſum is = the long. of nona. deg.	6	5	6	11

By the foregoing proceſs it appears, that the longitude of the nonageſimal degree is 6s 5° 6′ 11″, and the altitude 29° 38′ 12″.

APPENDIX.

APPENDIX.

THE aſcertaining of the difference of longitude between various places on the globe, particularly between important headlands and ſeaports, and alſo between places where aſtronomic obſervations are made, to perfect the knowledge of the heavens, and to improve navigation and geography, are very deſirable objects. Obſervations of eclipſes of the moon and of Jupiter's ſatellites have frequently been made uſe of for this purpoſe. When the beginning or ending has been obſerved in two places, the difference in time is the difference of meridians, which may be reduced to degrees, at the rate of 15° per hour. But the earth's ſhadow is ſo imperfectly defined at the moon, that the beginning or ending of a lunar eclipſe cannot be ſo ſatisfactorily determined, that the difference of meridians, deduced from theſe obſervations, can be entirely depended upon. The beginning or ending of the eclipſes of Jupiter's ſatellites can be determined with more preciſion; but ſtill, much depends upon the magnifying powers and the aperture of the teleſcopes, which are made uſe of for the obſervation: nor is the difference of meridians between two places, deduced from theſe obſervations, reckoned accurate, unleſs there is a conſiderable number, both of immerſions and emerſions, to be compared with correſponding ones. Of late years, therefore, obſervations of ſolar eclipſes, of occultations of fixed ſtars by the moon, and of tranſits of the inferior planets over the ſun's diſk, when made under favourable circumſtances, have been moſt ſought for, and uſed, when they could be obtained, for determining differences of meridians. In theſe obſervations, the difference of times is not the difference of meridians,

ridians, but is ſometimes more, and ſometimes leſs, according to the parallaxes. Theſe, therefore, muſt be carefully computed; and the tedíouſneſs of the proceſs has, doubtleſs, prevented obſervations of this kind from becoming ſo generally uſeful, as they would otherwiſe have been. But, as the difficulty may appear leſſened, by having the whole operation brought into one view, I now take the liberty of doing it, for the ſake of thoſe who have not made great advances in aſtronomy, ſhould any ſuch meet with this paper. I have taken, for examples, thoſe obſervations which have led to a determination of the difference of meridians between London and Cambridge, and Paris and Cambridge. An accurate determination of this difference may be very uſeful. Hereby, the aſtronomic tables, which have been fitted to the meridians of the celebrated obſervatories of Paris and Greenwich, are made our own, for all the purpoſes of calculation; and our obſervations may be made uſe of, for correcting and improving thoſe tables: And ſhould new ones be conſtructed for Cambridge, the ſame may be accommodated to the European obſervatories.

PROBLEM.

Given the moon's longitude, latitude, horizontal parallax and horizontal ſemi-diameter, together with the altitude of the nonageſimal degree of the ecliptic, and the angle of it's pole, or angle P*p*Z, and conſequently the longitude of the nonageſimal degree, to find her parallax in latitude and longitude, her viſible ſemi-diameter, or ſemi-diameter augmented agreeably to her altitude or zenith diſtance, and the viſible difference of longitude between the ſun and moon?

If the proceſs be trigonometrical we muſt firſt find the moon's altitude and parallactic angle.

In plate I, figure I, let the arc *pe*L cut the ecliptic in *e* = the moon's longitude, and let the arc *l a t* mark out her latitude; then the point of interſection of the two arcs at ☽, will be the moon's true place or poſition. Let the vertical arc Z☽N be drawn; then, in the triangle *p*☽Z, the ſide Z☽ will be the moon's zenith diſtance, or co-altitude, and the angle *p*☽Z her parallactic angle; to find which, there are given two ſides and the included angle, viz. the ſide *p*Z = the altitude of the nonageſimal degree; the ſide *p*☽ = the moon's diſtance from *p* one of the poles of the ecliptic = the complement of her latitude, and the angle Z*p*☽ = the difference between the moon's longitude and the longitude, of the nonageſimal degree.

From Z let fall the perpendicular Z*d*, dividing the oblique angled triangle *p*☽Z into two right-angled triangles, right-angled at *d*.

For ſide Z☽ the moon's zenith diſtance.

Radius : Sine Co-angle Z*p*☽ :: Tangent *p*Z : Tangent *pd*. Take *pd* from *p*☽ and the remainder will be ☽*d*; then ſay Sine Co-*pd* : Sine Co-☽*d* :: Sine Co-*p*Z : Sine Co-Z☽.

For angle *p*☽Z, the moon's parallactic angle.

Sine ☽*d* : Sine *pd* :: Tangent angle Z*p*☽ : Tangent angle *p*☽Z.

We muſt next find the moon's parallax in altitude, preparatory to which, let the moon's diſtance from the earth in ſemi-diameters of the earth be found.

In plate I, figure II, in the plain right-angled triangle B A C, right-angled at B, let C B be the ſemi-diameter of the earth, which call 1, the angle B A C the moon's horizontal parallax, the hypotenuſe C A the moon's diſtance from the earth's center in ſemi-diameters of the earth, to find which ſay Sine angle B A C : ſide B C :: Radius : hypotenuſe C A.

Having found the moon's diſtance from the earth's center, there are given in plate I, figure III, in the plain oblique-angled triangle B A C, the ſide A C = the moon's diſtance from the earth's center, the ſide B C = the earth's ſemi-diameter, and the included angle B C A = the moon's true zenith diſtance, to obtain angle B A C = the difference between angle B C A the true zenith diſtance and D B A the viſible zenith diſtance = the moon's parallax in altitude; to find which, ſubtract angle B C A from 180° the ſum of the three angles, the remainder will be the ſum of the two unknown angles A B C and B A C: Take the ſum and difference of the ſides A C and B C; then ſay, the ſum of the two ſides A C and B C : their difference :: the Tangent of half the ſum of the two unknown angles : the Tangent of half their difference. Subtract the half difference from the half ſum, and the remainder will be the leſſer angle BAC = the moon's parallax in altitude,* which added to the true zenith diſtance, will give the viſible = angle DBA.

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* The common method of deducing the moon's parallax in altitude from her true zenith diſtance is by approximation; finding the parallax for the true zenith diſtance as if it were the viſible, by adding together the logarithmic Sine of the zenith diſtance and of the horizontal parallax: then adding the parallax thus found to the true zenith diſtance, and conſidering the ſum as the viſible zenith diſtance, and then going over the work again; but I have exhibited the above method as the direct one; and it is but little longer than the other.

For the parallax in latitude and longitude.

Having found the moon's parallax in altitude ſuppoſe it ſet off plate I, figure I, upon the vertical arc from ☽ to m towards the horizon, becauſe the viſible zenith diſtance is greater than the true; then, the point m will be the moon's viſible place, or her place as ſeen from ſome given ſpot on the earth's ſurface. From p to L, through m, let the arc pmL be drawn; then, in the triangle pm☽, the angle mp☽ = the diſtance between n and e on the ecliptic is the moon's parallax in longitude; and the ſegment my, of the ſide pm, is her parallax in latitude; to find which by ſpheric trigonometry, there are given two ſides and the included angle, viz. ſide p☽, the moon's diſtance from the pole of the ecliptic; the ſide ☽m, the moon's parallax in altitude, and angle p☽m = the ſupplement of angle p☽Z to 180°. Or it may be more convenient to take the triangle L☽m; in which are given ſides L☽ and ☽m and the included angle L☽m, = p☽Z to obtain the ſame things. But the moon's parallax in latitude and longitude may be found more eaſily, by the following method.—Let fall the perpendicular mr upon the arc p☽L; then, there will be formed a right-angled triangle ☽mr, right-angled at r. As the perpendicular mr, in eclipſes of the ſun, and even in occultations of fixed ſtars by the moon, will always be near the ecliptic, it may be conſidered as parallal to it, without any ſenſible error; conſequently, the ſide mr may be reckoned = the ſegment ym. We may therefore call the ſide ☽r the moon's parallax in latitude, and the ſide mr, augmented in the ratio of the Sine of the moon's viſible co-latitude to radius,

dius, will be equal to ne, the parallax in longitude. In occultations, when the moon's latitude is conſiderable, this augmentation muſt take place; but in ſolar eclipſes, her latitude is always ſo ſmall, that the difference between mr and ne is imperceptible; therefore, mr may be conſidered as the true parallax in longitude.

As the ſides of the triangle ☽mr are very ſhort, it may be reckoned a plain one, without any perceptible error; and the ſides ☽r and mr may be found by plain right-angled trigonometry, the hypotenuſe ☽m, and the angle m☽r = angle p☽Z, being given.

For ſide ☽r, the moon's parallax in latitude.

Radius : ☽m in ſeconds :: Sine co-angle m☽r : ſide ☽r in ſeconds.

For ſide mr, the parallax in longitude.

Radius : ☽m in ſeconds :: Sine angle m☽r : ſide mr in ſeconds.

GENERAL RULES FOR APPLYING THE PARALLAXES.

In a place in north latitude, when the moon's latitude is north, if the parallax in latitude is leſs than the true latitude of the moon, ſubtract it from her latitude; the remainder will be her viſible latitude north. If the parallax be the greater, ſubtract her latitude therefrom; the remainder will be the moon's viſible latitude ſouth. If the true latitude of the moon be ſouth, the parallax in latitude, whether greater or leſs, muſt be added, and the viſible latitude will be ſouth. Theſe rules muſt be inverted, when the latitude of a place is ſouth.

The parallax in longitude muſt be added to the moon's true longitude, to give the viſible, when her true longitude is to the eaſt of the nonageſimal degree, otherwiſe, it is to be ſubtracted.

If the moon's true latitude and longitude are to be found from the viſible, the above rules, for applying the parallaxes, muſt be inverted.

For the moon's augmented or viſible ſemi-diameter according to her altitude or zenith diſtance.

The viſible diameter of the moon, to a ſpectator on the ſurface of the earth, is continually enlarging, from the horizon to the zenith; becauſe the ſemi-diameter of the earth bears a ſenſible proportion to the moon's diſtance. When the moon is in the zenith of an obſerver on the earth's ſurface, ſhe is nearer to him by almoſt a whole ſemi-diameter of the earth, than when ſhe is in his horizon. In the horizon, therefore, the moon's viſible diameter muſt be the leaſt, and in the zenith the greateſt of all.

The viſible diameter of any planet is inverſely as it's diſtance from us; therefore, to find the moon's augmented ſemi-diameter, from her horizontal, at a particular zenith diſtance, we muſt ſay,—The moon's diſtance from the earth's ſurface at the given viſible zenith diſtance : her horizontal diſtance, or diſtance from the earth's center† :: her horizontal ſemi-diameter : her augmented or viſible ſemi-diameter at the given zenith diſtance. But ſides of plain triangles are meaſured by the ſines of the oppoſite angles; thus in plate I, figure III, ſine angle BGA the moon's true zenith diſtance, meaſures the ſide AB, the moon's diſtance from a ſpectator on the earth's ſurface; and ſine angle A B C, or which is the ſame, ſine angle DBA, the moon's viſible zenith diſtance meaſures the ſide A C, the moon's diſtance from the earth's center = her horizontal diſtance. We may therefore ſay, Sine angle A C B, the moon's true zenith diſtance : ſine angle D B A, her viſible zenith diſtance :: her horizontal ſemi-diameter : her augmented or viſible ſemi-diameter.[*]

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† The moon's diſtance from a ſpectator at the earth's center is the ſame, at every altitude, as her horizontal diſtance.

For the viſible difference of longitude between the center of the ſun and moon.

To obtain this, there are given the ſum of the ſun's horizontal ſemi-diameter, and the moon's augmented or viſible ſemi-diameter, at the time of the obſerved beginning or ending of the eclipſe, for which the calculation is made, and the viſible latitude of the moon from the ſun, found by properly applying the parallax of latitude from the ſum, by which a plain right-angled triangle may be formed. Thus in the plain right-angled triangle ☉E☽, plate I, figure IV, there will be given the hypotenuſe ☉☽ = the ſum of the ſemi-diameter of the ſun or moon,[†] or viſible diſtance of their center, and the ſide E☽, the viſible difference of latitude between their centers, to find the ſide ⊙E, their viſible difference of longitude.* This may be obtained by Euclide, Book I, Prop. 47 ; or it may be found by common trigonometry thus ⊙☽ : Radius :: E☽ : ſine angle E⊙☽ ; then Radius : ⊙☽ :: ſine co-angle E⊙☽ = angle E☽⊙ : ⊙E. But ⊙E may be obtained more expeditiouſly, thus ; find the logarithms for the ſum and difference of ⊙☽ and E☽ ; the half ſum of theſe logarithms is the logarithm of ⊙E.

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[*] The augmentation of the moon's ſemi-diameter to her altitude or zenith diſtance may be found in ſome aſtronomical tables; but I thought it beſt to give the trigonometrical proceſs.

[†] This ſum of the ſemi-diameter ought to be diminiſhed, on account of diſfraction, or the inflection of the rays which paſs the limbs of the moon, ſuppoſed to be cauſed by her atmoſphere. When an eclipſe or occultation would appear to us to be juſt begun, were there no inflexion, this cauſes her limb to be viſibly diſtant from

Having found, for the place from whence we deſign to reckon the difference of meridians, the parallaxes, and the viſible difference of longitude between the centers of the ſun and moon, at the obſerved time of the beginning or ending of a ſolar eclipſe, or occultation of a fixed ſtar, or for both the beginning and ending, where both have been obſerved, we muſt aſſume the difference of meridians in time, between the firſt

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from the limb of the ſun, or from the fixed ſtar 4½″ : Therefore, at the inſtant when the beginning or end appears to us, it is obvious, that the centers cannot be ſo far aſunder by 4½″, as the ſum of the ſemi-diameters ; and by ſo much muſt this ſum be diminiſhed. For a brief account of this inflection, ſee ASTRONOMIE par M. DE LA LANDE, tome ii. art. 1992, 1993, edition ſeconde.

* As the ſun has a viſible latitude, equal to his parallax in latitude, the ſide ⊙E is not a portion of the ecliptic, but is parallel to it : But as it's utmoſt diſtance from it can never be more than between 8″ and 9″, the difference between this ſide and the viſible difference of longitude, is imperceptible. In an occultation, when the ſtar's latitude is large, this ſide, parallel to the ecliptic, muſt be enlarged, to give the viſible difference of longitude between the ſtar and the moon's center, in the ratio of the coſine of the moon's viſible latitude, or of the ſtar's latitude to Radius, according as the one or the other is neareſt to the ecliptic.

place, and that whoſe longitude is ſought; and muſt find the parallaxes the augmented ſemi-diameter of the moon and the difference of longitude between the centers of the ſun and moon, or ſtar and moon, for the obſerved time of the beginning or ending of the eclipſe, or occultation, at this ſecond place.

We are next to find the time of the true ecliptic conjunction of the moon and ſun, or moon and ſtar, according to obſervation: And here we are to obſerve, that at the beginning of an eclipſe or occultation, the moon is viſibly not ſo far advanced in longitude, as the ſun or ſtar, therefore, if we ſubtract the viſible difference of longitude from the ſun's viſible longitude,* or from the ſtar's true longitude,† we ſhall have the viſible longitude of the moon. If to this viſible longitude, we apply her parallax in longitude, with it's proper ſign, we ſhall have her true longitude according to obſervation, the ſun's or ſtar's longitude being ſuppoſed to be accurately given by the tables. If we make uſe of the moon's parallax in longitude from the ſun inſtead of her ſimple parallax, which will prevent a ſeparate calculation for the ſun's parallax in longitude, the viſible difference of longitude being ſubtracted from, or added to the ſun's true longitude will give the moon's viſible longitude nearly; and her parallax in longitude from the ſun properly applied to this will give her true longitude.

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* The ſun's viſible longitude is found by applying his parallax in longitude with the proper ſign to his true longitude.

† As the fixed ſtars have no perceptible parallax, there can be no diſtinction in calculations between the viſible and true longitude.

The difference between the ſun's or ſtar's longitude and the moon's thus found will, it is obvious, be the true difference of longitude between them. But this may be found more readily thus—In eclipſes, if the viſible difference of longitude and the moon's parallax in longitude from the ſun, and in occultations, the moon's ſimple parallax, be both additive or both ſubtractive, their ſum is the true difference of longitude, between the moon and ſun or ſtar. If the one is additive, and the other ſubtractive, their difference is the difference of longitude.

It ſometimes, though rarely, happens, that the moon, at the beginning of an eclipſe or occultation, has paſt the true conjunction,* although her viſible longitude is leſs ; this is the caſe when the parallax, being additive, is greater than the difference of longitude between the moon and ſun or ſtar. We muſt invert this rule, to know whether the moon has paſt the true conjunction, at the end.

The true difference of longitude between the moon and ſun, or moon and ſtar being found, both for the beginning and ending of a ſolar eclipſe, or of an occultation, where both have been determined by obſervation, the ſum of theſe, when the moon was on different ſides of the point of the true ecliptic conjunction, at the beginning and ending or the difference, when ſhe was upon the ſame ſide in both, will be her whole motion in longitude from the ſun or ſtar during the time of the

eclipſe

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* We ſhall find an inſtance of this at the beginning of the ſolar eclipſe at Greenwich, June 24, 1778.

eclipſe or occultation, according to obſervation.* Then we muſt ſay, This whole motion : the whole time of the duration :: the difference of longitude at the beginning or ending : the diſtance in time between the obſerved beginning or ending and the true ecliptic conjunction.

When the beginning or ending only has been obſerved, at the places for which we made the calculations, we muſt uſe the moon's horary motion from the ſun, as given by the tables, for the firſt term, and one hour or 3600″, for the ſecond term.

The time thus found is to be added to the obſerved time of the beginning of the eclipſe or occultation, when the moon is behind the true conjunction ; but if ſhe has paſſed it, then it muſt be ſubtracted ; and the ſum or difference will be the time of the true ecliptic conjunction according to obſervation.

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* If this whole motion is conſiderably different from that given by the tables, and we make dependance upon the obſervations of the beginning and end of the eclipſe or occultation, we are to conclude, that the moon's latitude, by the tables, is not exact ; and the correct latitude muſt be ſought, for the beginning and end. Theſe being obtained, the viſible difference of longitude between the ſun's and moon's center is to be found, conformably thereto. By this it is ſuppoſed, that the moon's horary motion, by the tables, is true ; which may be concluded to be generally the caſe, in eclipſes and occultations ; and indeed it is much more likely that the latitude ſhould be given too ſmall or too large, in the tables, than that the horary motion ſhould be conſiderably erroneous by them. In more than five hundred longitudes of the moon, calculated from Mayer's printed tables, by M. Lemery, which he compared with correſponding longitudes, deduced from the lunar obſervations of the late accurate and celebrated Dr. Bradley ; which compariſons are publiſhed in *Connoiſance des Temps pour l' Annèe*, 1783, I find that the error in the moon's horary motion in longitude rarely amounts to 2″. It is not common for it to exceed 1″ ; and it is generally but a few tenths of a ſecond. We may therefore make great dependance upon the horary motion given by theſe tables.

This rule is to be inverted, to find the time of conjunction from the obſervation of the end of the eclipſe or occultation.

The time of the true ecliptic conjunction, according to the obſervations, being found in this manner, for each place, the difference of times will be the difference of meridians.

Let the preceding rules be now exemplified.

The beginning of a ſolar eclipſe was obſerved at the Royal Obſervatory at Greenwich, by Dr. Maſhelyne, the Aſtronomer Royal, Auguſt 5, 1766, at $5^{h.}\ 29'\ 56''$ P. M. apparent time, and by his aſſiſtant at $5^{h.}\ 29'\ 58''$ P. M. the mean being $5^{h.}\ 29'\ 57''$. The end was obſerved by the Doctor at $7^{h.}\ 11'\ 27''$, and by his aſſiſtant at $7^{h.}\ 11'\ 40''$ P. M. the mean being $7^{h.}\ 11'\ 33''\frac{1}{2}$.

The ſame eclipſe was obſerved by the late Dr. Winthorp, Hollis Profeſſor of the Mathematics and Natural Philoſophy, at Cambridge, who made the beginning at $11^{h.}\ 39'\ 23''$ A. M. and the end at $2^{h.}\ 45'\ 9''$ P. M. apparent time.

Required the difference of meridians between Greenwich and Cambridge, by theſe correſponding obſervations?

The ſolar and lunar elements for calculating the parallaxes, &c. are from Mayer's tables.

For Greenwich at $5^{h.}\ 29'\ 57''$.

The ſun's longitude,	$4^{s.}$	13°	9′	51″	0
The moon's ecliptic longitude,	4	13	3	59	0

The

The moon's latitude north decreaſing,		33′	17″	0
The ſun's horary motion,		2	43	2
The moon's horary motion in longitude in the ecliptic,		29	29	5
The moon's horary motion in latitude,		2	43	2
The moon's horizontal parallax for Greenwich,		53	58	4
The ſun's horizontal parallax,			8	4
The moon's horizontal parallax from the ſun,		53	50	0
The ſun's horizontal ſemi-diameter by obſervation,		15	47	0
The moon's horizontal ſemi-diameter,		14	44	5
The obliquity of the ecliptic,	23	28	18	0
The latitude of Greenwich Obſervatory,	51	28	39	
Reduced to the center,	51	14	11	
Complement = PZ	38	45	49	

From theſe elements the altitude of the nonageſimal degree

has already been found,	s.	29°	38′	12″
And it's longitude,	6	5	6	11
From which ſubtract the moon's longitude,	4	13	3	59
The difference is equal to angle *Zp* ☽,	1	22	2	12
		= 52	2	12

For the moon's zenith diſtance and parallactic angle. Plate I.

Figure I.

For ſide Z ☽.

Radius,						
: Sine Co-angle	*Zp* ☽	37	57	48	9	7889864
:: Tangent	*pZ*	29	38	12	9	7550557
: Tangent	*pd*	19	17	20	9	5440421
	p ☽	89	26	43		
	☽*d*	70	9	23		

Sine

	Sine Co	*pd*	70	42	40	9	9749099
:	Sine Co	☽*d*	19	50	37	9	5307810
::	Sine Co	*p*Z	60	21	48	9	9391086
:	Sine Co	Z☽	18	12	56	9	4949797
			90				
☽'s zen. diſt.		= Z☽	71	47	4		

For angle *p*☽Z.

	Sine	☽*d*	70	9	23	9	9734755
:	Sine	*pd*	19	17	20	9	5189498
::	Tangent angle	Z*p*☽	52	2	12	10	1077631
:	Tangent angle	*p*☽Z	24	13	55	9	6532974
		= *m*☾*r* the parallactic angle.					

For the moon's diſtance from the earth in ſemi-diameters of the earth. Plate I. Fig. II.

	Sine angle BAC = the moon's horizontal parallax from ſun,	0° 53′ 50″		8	1947596
:	BC, the earth's ſemi-diameter,	1			
::	Radius			10	
:	AC, the moon's diſtance from the earth's center,	63	8617*	1	8052404

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* This is not ſtrictly the moon's diſtance from the earth, becauſe her horizontal parallax from the ſun is here uſed, inſtead of her true parallax; but the number thus found is to be taken for the next proceſs, becauſe, in theſe calculations, we find her parallax in altitude, latitude and longitude from the ſun, to prevent ſeparate proceſſes for his parallaxes. When we are making theſe calculations in occultations of the fixed ſtars by the moon, her true horizontal parallax will neceſſarily be uſed, becauſe the fixed ſtars have no diſcoverable parallax.

For the moon's parallax in altitude. Plate I, Fig. III.

Sum of the three angles,	180° 0′ 0″
Angle BCA = ☽'s zenith diſtance,	71 47 4
Sum of the two unknown ∠'s	180 12 56
Half ſum	54 6 28

Side CA = ☽'s diſtance from the earth's center,	63 8617
Side BC = earth's ſemi-diameter,	1
Sum of the two ſides including the known angle,	64 8617
Their difference,	62 8617

The ſum of the two ſides including the known angle,	64 8617	1 8119883
: Their difference,	62 8617	1 7983861
:: Tangent ½ ſum of the two unknown angles,	54° 6′ 28″	10 1404579
: Tangent ½ their difference,	53 15 5	10 1268557
Leſſer angle BAC or ☽'s parallax in altitude from ſun,	51 23	
	60	
	3083	

The finding of the moon's parallax in altitude, from her true zenith diſtance, may be much facilitated by the following ſhort table, which I have calculated for the purpoſe.

TABLE.

☽'s horiz. par^{x}.	Logarithms.	Difference.
53′	0 0133919	2526
54	0 0136445	2526
55	0 0138971	2526
56	0 0141497	2527
57	0 0144024	2527
58	0 0146551	2527
59	0 0149078	2528
60	0 0151606	2528
61	0 0154134	2528
62	0 0156662	

The firſt column of this table contains the moon's horizontal parallaxes in minutes. In the ſecond column are the logarithms, which are to be ſubtracted from the logarithmic tangent of the half ſum of the two unknown angles (found as above) according to the moon's horizontal parallax. The remainder will be the logarithmic tangent of the half difference of thoſe angles. The degrees, &c. anſwering to this, being ſubtracted from the half ſum, will give the parallax in altitude.

The uſe of the table exemplified.

Required the moon's parallax in altitude from the ſun, her horizontal parallax from the ſun being 53′ 50″, and her true zenith diſtance 71° 47′ 4″, as before?

The Tang. ½ ſum of the two unknown angles, as above,	54° 6′ 28″	10 1404579
Logarithm from the table for	53 50	0 0136024
The Tang. ½ difference of the two unknown angles,	53 15 5	9 1268555
The moon's parallax in altitude from the ſun, as before,	51 23	

For the moon's parallax in altitude from the ſun by approximation.

Sine of the moon's true zenith diſtance,	71° 47′ 4″	9 9776721
The moon's horiz. par. from the ſun,	3230*	3 5092025
Moon's par. in altitude from ſun nearly	3068 = 51′ 8″	3 4868746
	71 47 4	
The moon's viſible zen. diſt. nearly	72 38 12 Sine	9 9797448
	3230″ Log.	3 5092025
The moon's par. in alt. from the ſun,	3082, 8	3 4889473

This is but $\frac{2}{10}$ of a ſecond different from the parallax which was found by the direct method.

For the moon's parallax from the ſun in longitude and latitude.
Plate I, Figure I.

Radius		
: ☽*m* the moon's parallax in altitude from the ſun,	3083″	3 4889735
:: Sine angle $p☽Z = m☽r$ the parallactic angle,	24° 13′ 55″	9 6132407
: *mr* the moon's parallax in longitude from the ſun,	1265, 3 =21′ 5″, 3	3 1022142

Radius		
: ☽*m*	3083″	3 4889735
		:: Sine

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* The arc of this parallax is ſo ſmall, that we may uſe the logarithm of the ſeconds contained in it, inſtead of the ſine of the arc without any ſenſible error.

:: Sine co-angle p☽Z	65°46′ 5″	9 9599432
: ☽r the moon's paral. in lat. from ſun,	2811, 3	
	= 46′ 51″, 3	3 4489167

For the moon's viſible or augmented ſemi-diameter.

The Sine of the moon's true zen. diſt.	71 47 4	9 9776721
: The Sine of her viſible zen. diſt.	72 38 27	9 9797546
:: Her horiz. ſemi-diameter, 14′ 44″, 5 =	884″, 5	2 9466978
: Her augmented ſemi-diameter,	888, 8	
		2 9487803

Such is the trigonometrical proceſs, for finding the moon's parallax in latitude and longitude, and her augmented or viſible ſemi-diameter; but aſtronomers have invented *formulas*, which greatly ſhorten the work, making it unneceſſary to find the moon's zenith diſtance, parallactic angle, and parallax in altitude. In ſolar eclipſes, the moon's latitude being very ſmall, the *formulas* are quite conciſe and eaſy.

Let the parallax in latitude be called x, and in longitude y. Let the co-ſine of the altitude of the nonageſimal degree be called C and it's ſine D. Let the moon's horizontal parallax or her horizontal parallax from the ſun be called p, her *viſible* latitude l, and *true* latitude L. Let the diſtance between the moon's *viſible* longitude and the longitude of the nonageſimal degre $= Zpm$ in Plate I, Fig. I, be called b; the *true* $= Zp$☽, B. Then, the *viſible* latitude and longitude of the moon being given,

$x = pC \mp pD$ ſine l co-ſine b,

$y = pD$ ſine b.

The ſubtractive ſign — is to be uſed, when the moon's latitude is north, the additive + when it is ſouth.

In occultations, when the moon's latitude is large,

$x = p\text{C co-ſine } l \mp p\text{D ſine } l \text{ co-ſine } b;$

$y = \dfrac{p\text{D ſine } b}{\text{Co-ſine L}}$ } i. e. *y*, found by *p*D ſine *b*, muſt be enlarged in ratio of the co-ſine of the moon's true latitude to Radius.

Formulas have been invented to find the parallaxes in a direct way, when the moon's *true* latitude and longitude are given; but it is rather eaſier, in this caſe, to come at them by approximation, thus,

Add together logarithm *p*, ſine D, ſine B, which will give *y*, or the parallax in longitude nearly. Add the parallax thus found to B, which will give *b* nearly, viz. the diſtance between the moon's *viſible* longitude and the longitude of the nonageſimal degree. Then log. *p*, ſine D, ſine *b* thus nearly found, will give *y*, the parallax in longitude, ſufficiently exact; and this added to B will give correct *b*. Add together log. *p*, ſine C, which will give *x*, or the parallax in latitude nigh the truth. This being properly applied to L, the moon's *true* latitude, will make *l*, the *viſible* latitude, nearly. Then add together log. *p*, ſine D, ſine *l*, thus nearly found, and co-ſine *b*, which give the logarithm of a number, that added to, or ſubtracted from *x*, as before nearly found, will make the correct parallax in latitude.

With reſpect to the moon's augmented ſemi-diameter we may ſay B : C :: the moon's horizontal ſemi-diameter : her augmented

mented ſemi-diameter. Her augmented ſemi-diameter, thus found, will be ſufficiently correct for all purpoſes.

Exemplification.

The co-altitude of the nonageſimal degree,	60°21′48″	Sine	9 9391086	C
The altitude of the nonageſimal degree,	29 38 12	Sine	9 6941647	D
The moon's horiz. par. from the ſun,	53′50″ = 3230″	Log.	3 5092025	p
The diſt. between the moon's true long. and the long. of the nonageſimal degree,	52 2 12	Sine	9 8967492	B
The moon's true latitude,	33 17 N = L			

p 3 5092025
D 9 6941970
B 9 8967492

y nearly 1259″ 3 1001487
= 20′ 59
B 52 2 12

b nearly 52 23 11 Sine 9 8988045
pD 3 2033995

1265″, 3 3 1022040
= 21′ 5″, 3 ☽'s parallax in long. from ☉
B 52 2 12

b correct 52 23 17 Sine 9 8988143

p 3 5092025
C 9 9390986

x nearly 2807'',4   3 4483011
= 46' 47''   S
☽'s true lat. = L 33 17   N
+
l nearly, viz. ☽'s viſ. lat. 13 30 S Sine 7 5940588
pD 3 2033995
Co-ſine b 9 7855507

3'', 8   0 5830090
x nearly 46' 47, 4
+ 3, 8
x correct 46 51, 2 ☽'s parallax in lat. from ☉

	Sine B	52° 2' 12''	9 8967492
:	Sine b	52 23 17	9 8988143
::	The moon's horiz. ſemi-diameter,	14' 44'', 5 = 884'', 5	2 9466978
:	Her augmented or viſible ſemi-diameter,	888, 8 = 14' 48'', 8	2 9487629

By theſe reſults it appears that the foregoing *formulas* find the parallaxes and augmented ſemi-diameter of the moon to great exactneſs.

Let us now find the viſible difference of longitude between the center of the ſun and moon, at Greenwich, at 5h. 29' 57'', when the eclipſe begun there. Plate I, Fig. IV.

The

The ☉'s horizontal ſemi-diameter,	15′ 47″, 0
The ☽'s augmented ſemi-diameter,	14 48, 8
Sum,	30 35, 8
Subtract for inflection,	4, 5
Viſible diſtance of centers,	30 31, 3
	60
Hypotenuſe ☉☽	1831, 3
The ☽'s parallax in latitude from ☉,	46′ 51″, 2 S
The moon's latitude, —	33 17, 0 N
The ☽'s viſible latitude from ☉,	13 34, 2 S
	60
Side E☽	814, 2

Hypotenuſe ☉☽	1831, 3	
Side E☽±	814, 2	
Sum,	2645, 5	3 4225078
Difference,	1017, 1	3 0073637
		2 | 6 4298715
Side ☉E = the viſible diff. long. of ☉'s and ☽'s center,	1640, 3	3 2149357
	= 27 20, 3	

For the moon's true longitude according to obſervation.

From the ſun's true long. aug. $5^{D.}$ $5^{h.}$ 29′ 57″	$4^{S.}$ 13° 9′ 51″, 0
Subtract ☉'s and ☽'s viſible diff. long. by obſervation,	27 20, 3

Remains

Remains the ☽'s viſible longitude nearly,	4	12	42	30, 7
Add the moon's parallax in longitude from the ☉.			+ 21	5, 3
The ſum is the ☽'s true long. according to obſerv.	4	13	3	36, 0
The true diff. of long. of ☉ and ☽ according to obſerva.			6	15
			60	
			375	

Let us now find the requiſites for $7^{\text{h.}}$ 11′ 33½″ when the eclipſe ended at Greenwich.

Elements for computing the parallaxes, &c. at $7^{\text{h.}}$ 11′ 33½″

The ſun's longitude,	4s.	13°	13′	54″, 5
The moon's ecliptic longitude,	4	13	53	55, 6
The moon's latitude north,			28	40, 6
The ſun's right aſcenſion,		135	42	33
The right aſcenſion of the mid-heaven,		243	35	55
Angle *p*PZ,		26	24	5
Complement,		63	35	55

Side PZ as before, as alſo the ☽'s horizontal parallax.*

From theſe elements are found

The altitude of the nonageſimal degree,		20°	11′	5″
Angle P*p*Z,		126	12	38
The longitude of the nonageſimal degree,	7s.	6	12	38

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* As the moon was near her apogee, the horizontal parallax altered ſo very little, in a few hours, that the ſame may here be taken for the end, as for the beginning. If the moon is not near her apogee or perigee, at the time of a ſolar eclipſe, allowance ſhould be made for the alteration of the horizontal parallax, between the beginning and end.

Angle Z*p*☽ = B, 82°18′42″

The moon's parallax in longitude from the ſun, 18 25, 3

The moon's parallax in latitude from the ſun, 50 32, 5 S

The moon's augmented ſemi-diameter, 14 45, 1

The ſum of the ☉'s and ☽'s ſemi-diameters—4″,5 for inflection = C☽ } 1827,6″

The moon's viſible latitude from the ☉ = E☽ 1311, 9 S

The viſible diff. of long. of the ☉'s & ☽'s centers = ☉E 1272, 4

= 21′ 12″, 4

To the ſun's true longitude at $7^{h.}\,11'\,33''\tfrac{1}{2}$ $4^{s.}$ 13°13′ 54″,5

Add the viſible diff. of long. of ☉ and ☽ by obſerv. + 21 12, 4

The ſum is the ☽'s viſible longitude nearly, 4 13 35 6, 9

Add the ☽'s parallax in long. from the ſun, + 18 25, 3

The ſum is the ☽'s true long. according to obſ. 4 13 53 32, 2

The true diff. of long. of ☉ and ☽ at the end by obſ. 39 37, 7

60

= 2377, 7

The true diff. by obſervation at the beginning, + 375, 0

The whole motion of the ☽ from the ☉ in the ecliptic by obſervation, } 2752, 7

By Mayer's tables

The moon's long. at the beginning of the eclipſe, $4^{s.}$ 13° 3′ 59″,0

Ditto at the end, 4 13 53 55, 6

The difference = the moon's whole motion during the eclipſe, } 49 56, 6

The ſun's motion for the ſame time,	—4′ 3,″ 5
The whole motion of the ☽ from the ☉ by Mayer's tables	45 53, 1
	60
	2753, 1

By this it appears, that the whole motion of the moon from the ſun in the ecliptic, by obſervation, differs but 0″, 4 from that given by the tables: Therefore, if we ſuppoſe the horary motions by theſe to be correct, the latitude will be nearly exact, the obſervations being allowed good.

For the time of the true conjunction of the ſun and moon according to obſervation.

The beginning of the eclipſe by obſ. at Greenwich,	5^{h} 29′ 57″
The end,	7 11 33½
The difference = the duration	1 41 36½
	60
	101
	60
	6096½

The whole motion of ☽ from the ☉ during the eclipſe,	2752″, 7	3 4397589
: The time of the duration,	6096½,	3 7850806
:: Diff. long. ☽ & ☉ at the beginning by obſ.	375, 0	2 5740313
: The time from the begin. to ecliptic conju.	830½	2 9193530
	= 13′ 50″½	

To

To the time of the beginning of the eclipſe by obſerv.	$5^{h.}\ 29'\ 57''$
Add	$13\ \ 50\frac{1}{2}$
The ſum is the time of ecliptic conjunction by obſerv.	$5\ \ 43\ \ 47\frac{1}{2}$

SCHOLIUM.

From the obſervation of the beginning or ending of a ſolar eclipſe, at a place, for which aſtronomic tables have been conſtructed, or, at a place, whoſe difference of longitude from that, for which the tables have been conſtructed, is accurately aſcertained, the error in the moon's longitude as given by them, may be known, if they have given her latitude exact, or the error, if there be one, has been corrected by obſervation. For example, when the foregoing eclipſe began at Greenwich, the moon's longitude, by Mayer's printed tables, was $4^{s.}\ 13^{\circ}\ 3'\ 59'',0$, but, deduced from the obſervations, it was but $4^{s.}\ 13^{\circ}\ 3'\ 36'',0$; the error of theſe tables, therefore, in the moon's longitude, at that time, was $+\ 23''$, ſuppoſing the latitude exact. By the obſervation of the end the error was $23'',4$.

Having found the time of the ecliptic conjunction of the ſun and moon, at Greenwich, as deduced from the obſervations of the beginning and ending of the eclipſe, we are next to find the conjunction at Cambridge, from the obſervations made there.

Let us aſſume the difference of meridians between Greenwich and Cambridge to be $4^{h.}\ 44'\ 25''$, then, at $11^{h.}\ 39'\ 23''$, A. M. when the eclipſe began at Cambridge, it was $4^{h.}\ 23'\ 48''$, P.M. at Greenwich; and the ſun's longitude and right aſcenſion, and the moon's latitude and longitude were the ſame, at each place.

Having found theſe for Greenwich, at $5^{h.}\ 29'\ 57''$, P. M. it will be eaſy to obtain them for $4^{h.}\ 23'\ 48''$, anſwering to $11^{h.}\ 39'\ 23''$ at Cambridge, by the ſolar and lunar motions for $1^{h.}\ 6'\ 9''$, the difference between $5^{h.}\ 29'\ 57''$ and $4^{h.}\ 23'\ 48''$.

Elements for Cambridge at $11^{h.}\ 39'\ 23$, A. M.

The ſun's longitude,	4s.	13°	7′	12″,5	
The moon's ecliptic longitude,	4	12	31	28,1	
The moon's latitude north,			36	16,8	
The moon's horizontal parallax,			54	0,5	
The ſun's right aſcenſion,		135	35	35	
The right aſcenſion of the mid-heaven,		130	26	20	
Angle *p*PZ		139	33	40	
=		40	26	20	
Complement		49	33	20	
The latitude of Cambridge,		42	23	28	N
Reduced to the center,		42	8	37	
Complement = PZ		47	51	23	

From theſe elements are found,

The altitude of the nonageſimal degree,		67	0	6	
Angle P*p*Z		31	30	11	
The longitude of the nonageſimal degree,	4	1	30	11	
Angle Z*p*☽ = B		11	1	17	
The moon's parallax in longitude from the ſun,			9	36,8	
The moon's parallax in latitude from the ſun,			20	49,9	S
The moon's augmented ſemi-diameter,			14	57,2	
The ſum of the ☉'s and ☽'s ſemi-diameter—4″,5 for inflection = ☉☽,			18	39,7	
The moon's viſible latitude from the ſun = E☽,			9	26,9	N

The viſible diff. of long. of the ⊙'s & ☽'s centers = ⊙E 1589'',1
= 26' 29'',1

From the ſun's true longitude at Cambridge, at 11$^{h.}$ 39' 23'',	4$^{s.}$ 13° 7 12, 5
Subtract ⊙'s and ☽'s viſible diff. long. by obſervation,	— 26 29, 1
	4 12 40 43, 4
Subtract the moon's parallax of longitude from ⊙,	— 9 36, 8
The moon's true long. according to obſerva.	4 12 31 6, 6
The true diff. of long. of ⊙ & ☽ according to obſerva.	36 5, 9
	60
	2165, 9

Elements for the end of the eclipſe at Cambridge, at 2$^{h.}$ 45' 9'', P. M.

The ſun's longitude,	4$^{s.}$ 13° 14' 37'',7
The moon's ecliptic longitude,	4 14 2 46, 7
The moon's latitude north,	27 51, 5
The ſun's right aſcenſion,	135 43 1
The right aſcenſion of the mid-heaven,	177 0 16
Angle *p*PZ	92 59 44
	= 87 0 16 Complement

From theſe elements are found,

The altitude of the nonageſimal degree,	53 7 38
Angle P*p*Z,	67 45 32
The longitude of the nonageſimal degree,	5 7 45 32

Angle

Angle Z*p*☽ = B,	23°42′45″
The moon's parallax in longitude from the ſun,	17 31, 7
The moon's parallax in latitude from the ſun,	32 22, 5
The moon's augmented ſemi-diameter,	14 54, 7
The ſum of the ☉'s and ☽'s ſemi-diameters — 4″,5 for inflection = ☉☽,	1837, 2
The moon's viſible latitude from the ☉ = E☽,	271, 8
The viſible diff. of long. of ☉'s and ☽'s centers = ☉E	1817, 1
	= 30 17, 1
To the ſun's true longitude at Cambridge, at $2^{h.}$ 45′ 9″,	$4^{s.}$ 13 14 37, 7
Add ☉'s and ☽'s viſible diff. long. by obſervation,	+ 30 17, 1
	4 13 44 54, 8
Add the moon's parallax of longitude from ☉,	+ 17 31, 7
The true long. of the moon according to obſ.	4 14 2 26, 5
The true diff. of long. of ☉ and ☽ according to obſerva.	47 48, 8
	60
	2868, 8
The true difference by obſervation at the beginning $11^{h.}$ 39′ 23″, A. M.	2165, 9
The whole motion of ☽ from ☉ in the ecliptic by obſ.	5034, 7

By Mayer's tables,

The moon's long. at the beginning of the eclipſe,	$4^{s.}$ 12°31′28″,1
Ditto at the end,	4 14 2 46, 7
Diff. = ☽'s whole motion in the ecliptic during the eclipſe,	1 31 18, 6

The ſun's motion for the ſame time, 7' 25'',2

The whole motion of ☽ from ⊙ by Mayer's tables, 1 23 53, 4
60
83
60
5033, 4

By this it appears that the whole motion by obſervation differs but 1'',3 from that given by Mayer's tables.

The time of the duration of the eclipſe by obſervation was 11146''.

For the time of the true conjunction of the ſun and moon at Cambridge by obſervation.

The whole motion of ☽ from ⊙ in the ecliptic during the eclipſe, 5034'',7 3 7019736
: The time of the duration, 11146 4 0471190
:: Diff. long. ☽ and ⊙ at the begin. by obſerv. 2165, 9 3 3356384
: The time from the beginning to the ecliptic ☌ by obſervation, 4795 3 6807838
= 79' 55''
Add this time, = 1h. 19 55
To the time of the beginning by obſerva. 11 39 23
The ſum is the time of ecliptic ☌ by obſerva. 0 59 18
Time of ecliptic ☌ at Greenwich by obſerva. 5 43 47½
Diff. = the diff. of meridians between Greenwich & Cambridge by obſerva. 4 44 29½

Thus it appears, that according to the obſervations of this eclipſe made at Greenwich and Cambridge, the difference of meridians between thoſe places is 4h. 44′ 29½″; ſo that the time that was aſſumed for the difference was very near the truth. If the difference of meridians, thus deduced, is in any caſe conſiderably different from that which was aſſumed, this reſult is to be conſidered as only the approximate difference, and is to be uſed as a new-aſſumed difference, from which deductions are to be made for the true. If the firſt reſult does not differ more than two or three minutes from the aſſumed difference of meridians, the ſame parallax in longitude and latitude may be uſed in the ſecond proceſs, which were found and uſed in the firſt; becauſe the right aſcenſion of the mid-heaven will not differ more than 6″ or 8″ from the firſt = the difference in the ſun's right aſcenſion, which, together with the ſmall difference in the moon's longitude and latitude, will make ſo trifling a difference in her zenith diſtance and parallactic angle, that the difference of parallaxes will be inconſiderable. All, therefore, that is neceſſary to be done, in the ſecond proceſs, is to find the ſun's longitude and the moon's latitude for the approximate or new aſſumed difference of meridians, from whence, with the parallaxes, we muſt deduce the difference of longitude between the ſun and moon, whereby the true conjunction will be obtained.

For illuſtration, let us take the foregoing obſervations of the ſolar eclipſe, and aſſume 4h. 42′ 25″ for the difference of meridians between Greenwich and Cambridge.

According to this aſſumption, at 11h. 39′ 23″, A. M. when the eclipſe began at Cambridge,

The

The ſun's longitude by the tables,	4ˢ· 13° 7′ 7″,7
The moon's ecliptic longitude,	4 12 30 29, 1
The moon's latitude,	36 22, 2
The ſun's right aſcenſion,	135 35 30
The right aſcenſion of the mid-heaven,	130 26 15
Angle *p*PZ	139 33 45
=	40 26 15
Complement	49 33 45

Hence

The altitude of the nonageſimal degree,	67 0 7
Angle P*p*Z,	31 29 49
The longitude of the nonageſimal degree,	4 1 29 49
Angle Z*p*☽ = B,	11 0 40
The moon's parallax in longitude from the ſun,	9 36, 3
The moon's parallax in latitude from the ſun,	20 49, 8 S
⊙ ☽ as before,	1839″,7
E☽ or the moon's viſible latitude from the ſun,	932, 4
The viſible diff. of long. of ⊙'s and ☽'s centers = ⊙E	1585, 9
	= 26′ 25″,9
The ☽'s ecliptic long. by obſerva. according to the above aſſumption,	4ˢ· 12° 31 5, 5
The true diff. of long. of ⊙ and ☽ by obſervation according to the aſſumption,	36 2, 2
	60
	2162, 2

At the end of the eclipſe at Cambridge at 2ʰ· 45′ 9″ according to the above aſſumption,

The ſun's longitude,	4^{s}	13°	14′	32″,9
The moon's ecliptic longitude,	4	14	1	47, 7
The moon's latitude,			27	56, 9
The ſun's right aſcenſion,		135	42	56
The right aſcenſion of the mid-heaven		177	0	11
Angle *p*PZ,		92	59	49
		= 87	0	11
Complement		2	59	49

Hence

The altitude of the nonageſimal degree,		53	7	40
Angle P*p*Z		67	45	30
The longitude of the nonageſimal degree,	5	7	45	30
Angle Z*p*☽ = B		23	43	42
The moon's parallax in longitude from the ſun,			17	31, 7
The moon's parallax in latitude from the ſun,			32	22, 4
☉☽ as before,				1837″,2
E☽				265, 5
The viſible diff. of long. of ☉'s and ☽'s centers = ☉E				1817, 9
				= 30′ 17″,9
The moon's ecliptic long. by obſervation according to the above aſſumption,	4	14	2	22, 5
The true diff. of long. of ☉ and ☽ by obſerva. at the end of the eclipſe, according to the aſſumption,			47	49, 6
			60	
				2869, 6
The difference at the beginning as above,				2162, 2
The whole motion of ☽ from ☉ accord. to this aſſum.				5031, 8
				5031″,

$5031'', 8 : 11146'' :: 2162'', 2 : 4789\frac{1}{2}'' = 1^{h.}\ 19'\ 49\frac{1}{2}''$

The beginning of the eclipſe at Cambridge, 11 39 23

The ecliptic ☌ of ☉ and ☽ by obſerva. according to the foregoing aſſumption, 0 59 12½

Time of ☌ at Greenwich by obſervation, 5 43 47½

The difference of meridians by this aſſumption, 4 44 35

By this it appears, that the aſſumed difference of meridians, viz. $4^{h.}$ 42′ 25″, differs much from the truth : But the parallaxes reſulting from this aſſumption, are ſo very near thoſe reſulting from the firſt, that it is evident they may be employed in the remaining part of the proceſs, without cauſing any ſenſible error. Let $4^{h.}$ 44′ 35″, the difference of meridians now found, be uſed as a new-aſſumed difference, which will give the moon's latitude at the beginning of the eclipſe at Cambridge 36′ 16″, 5, and at the end 27′ 51″, 2, and by a proper proceſs we ſhall find the viſible difference of longitude of ☉ and ☽ at the beginning 1589″, 3, and the true difference 2165″, 6 ; and at the end the viſible difference 1817,″ 1, and the true 2863″, 8. Hence we find the time of ecliptic conjunction at Cambridge $0^{h.}$ 59 17½″, and the difference of meridians between Cambridge and Greenwich $4^{h.}$ 44′ 30″, but half a ſecond different from what it was at firſt found.

If inſtead of finding the difference of meridians at once, by the conjunction at each place, any one chuſes to find it by the beginning and end ſeparately, he may eaſily do it.

At the beginning of the foregoing eclipſe at Greenwich, it appears, that the moon's ecliptic longitude by obſervation was 4$^{s.}$ 13° 3′ 36″, 0, but when it began at Cambridge, her longitude by obſervation was but 4$^{s.}$ 12° 31′ 6″. The difference is 32′ 29″, 4 = 1949″, 4. Let us find how long the moon was in paſſing over this portion of the ecliptic, thus,

The ☽'s horary mot. in long. 29′29″,5 =	1769″,5	3 2478506
: 1 hour,	3600	3 5563025
::	1949, 4	3 2899010
:	3966	3 5983529
	= 1$^{h.}$ 6′ 6″	

By this it appears, that it was 1$^{h.}$ 6′ 6″ after the eclipſe began at Cambridge, before the moon had the ſame longitude, as when the beginning was obſerved at Greenwich : Therefore, to 11$^{h.}$ 39′ 23″, A. M. add 1$^{h.}$ 6′ 6″ ; the ſum 0$^{h.}$ 45′ 29″, ſhews the time at Cambridge, when ſhe had the ſame longitude as at Greenwich at 5$^{h.}$ 29′ 57″ ; and the difference 4$^{h.}$ 44′ 28″ is the difference of meridians in time, between thoſe two places as deduced from the obſervations of the beginning of this eclipſe. By a ſimilar proceſs, we find the difference reſulting from the obſervations of the end to be 4$^{h.}$ 44′ 31½″. The mean is 4$^{h.}$ 44′ 29¾″.

Another ſolar eclipſe, the obſervations of which we make uſe of, to determine the difference of meridians between Greenwich and Cambridge, is that of June 24, 1778. As the clouds were troubleſome, at that time, at Cambridge, and conſequently the obſervations ſomewhat uncertain, we may take the obſervation of the end of the eclipſe made at Chelſea, by the Rev. Mr. Payſon, who had a very favourable time. And as Mr. Payſon and I have, by terreſtrial meaſurement, made with much preciſion, found Chelſea to be 26′′ in time eaſt of Cambridge, this obſervation is the ſame to our purpoſe, as if made in Cambridge.

The beginning of this eclipſe was obſerved by the Rev. Dr. Nevil Maſkelyne, at the Royal Obſervatory at Greenwich, at $3^{h.}\ 40'\ 11''$, and the end at $5^{h.}\ 25'\ 12''$ apparent time.* At Chelſea, the end was obſerved by the Rev. Phillips Payſon, at his own houſe, at $11^{h.}\ 38'\ 23''$, A. M. apparent time.

At the Royal Obſervatory at Greenwich, at $3^{h.}\ 40'\ 11''$ apparent time.

The ſun's longitude by Mayer's tables,	$3^{s.}\ 3^{\circ}\ 4'\ 2'',0$
The moon's ecliptic longitude,	3 3 6 32, 0

The

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* I am indebted to Mr. De La Lande, a celebrated member of the Royal Academy of Sciences at Paris, for the European obſervations of this eclipſe ; who, in a very obliging manner, in a letter of November 30, 1781, communicated them to me. They are as follow :

	At Oxford,	Greenwich,	Deptford,	Stockholm,	Paris,
Beg.	$3^{h.}\ 33'\ 44''$	$3^{h.}\ 40'\ 11''$	$3^{h.}\ 40'\ 12''$	$5^{h.}\ 4'\ 19''$	$3^{h.}\ 53'\ 15''$ or $13''$
End.	5 19 48	5 25 12	5 25 $11\frac{1}{2}$	6 13 25	

	Marſeilles,	Padua,	Milan,	Cadiz,	Toulouſe,
Beg.	4 12 0	4 41 46	4 29 9	3 18 53	3 52 27 or 24 or 22
End.	6 1 46	6 21 41	6 12 0	5 26 25	

As the beginning only was obſerved at Paris, I have taken the obſervations made at Greenwich, where the end as well as the beginning of the eclipſe was ſeen.

The moon's latitude north increaſing,	19′ 20″,7
The ſun's horary motion,	2 23, 0
The moon's horary motion in long. in the ecliptic,	37 36, 3
The moon's horary motion in latitude,	3 9, 8
The moon's horizontal parallax,	61 7, 6
The ſun's horizontal parallax,	8, 4
The ſun's right aſcenſion,	93°20 36
The right aſcenſion of the mid-heaven,	148 23 21
Angle *p*PZ,	121 36 39
	= 58 23 21
Complement	31 36 39
The ſun's ſemi-diameter,	15 47
The moon's horizontal ſemi-diameter,	16 40, 3
The obliquity of the ecliptic,	23 28 6
PZ, &c. as before,	

Hence

The altitude of the nonageſimal degree,	54 13 45
Angle P*p*Z,	41 5 7
The longitude of the nonageſimal degree,	4 11 5 7
Angle Z*p*☽ = B,	37 58 35
The moon's parallax in longitude from the ſun,	30 47, 6
The moon's parallax in latitude from the ſun,	35 50, 1
The moon's augmented ſemi-diameter,	16 51, 8
The ſum of 's & ☽'s ſemi-diam.—4 ′,5 for infl. = ⊙☽,	1954″,3
The moon's viſible lat. from the ſun = E☽	989, 4
The viſible diff. of long. of ⊙'s and ☽'s centers ⊙E,	1685, 3
	= 28 5″,3
The moon's ecliptic long. by obſervation,	$3^{s.}$ 3° 6 44, 3

At the Royal Obſervatory at Greenwich, at $5^{h.}$ 25′ 12″ apparent time.

The ſun's longitude,	$3^{s.}$ 3° 8′ 12″,2
The moon's ecliptic longitude,	3 4 12 21, 0
The moon's latitude north,	42 52, 8
The ſun's right aſcenſion,	93 25 8
The right aſcenſion of the mid-heaven,	174 43 8
Angle *p*PZ,	95 16 52
	= 84 43 8
Complement	5 16 52

Hence

The altitude of the nonageſimal degree,	46 11 19
Angle P*p*Z	59 45 50
The longitude of the nonageſimal degree,	4 29 45 50
Angle Z*p* ☽ = B,	55 33 29
The moon's parallax in longitude from the ſun,	36 33, 5
The moon's parallax in latitude from the ſun,	42 20, 7
The moon's augmented ſemi-diameter,	16 47, 8
The ſum of ☉'s & ☽'s ſemi-diam.—4″,5 for infl. = ☉☽,	1950″,3
The moon's viſible lat. from the ſun = E☽,	1047, 9
The viſible diff. of long. of ☉'s and ☽'s centers = ☉E,	1644, 9
	= 27′ 24″, 9
The moon's ecliptic long. by obſervation,	$3^{s.}$ 4° 12 10, 6

Hence it appears, that the motion of the moon by obſervation, during the eclipſe, with reſpect to the ecliptic, was 1° 5′ 26″,3; but, by Mayer's tables, it was 1° 5′ 49″. Suppoſing then the horary motion by the tables to be true, as is moſt probable, we may conclude, that there is an error in the moon's latitude, as given by them, the correction of which may be found, in the manner following :

The ☽'s longitude at the beginning, per tables,	3s. 3° 6′ 32″,0
The ☽'s parallax in longitude from ☉	— 30 47, 6
	3 2 35 44, 4
The ☽'s longitude at the end per tables,	3 4 12 21, 0
The ☽'s parallax in longitude from ☉	— 36 33, 5
	3 3 35 47, 5
Subtract	3 2 35 44, 4
Difference	1 0 3, 1
Subtract ☉'s motion during the eclipſe,	4 10, 2
Remains ☽'s viſible motion from ☉ per tables,	55 52, 9
	60
	3352, 9
The ☽'s viſi. lat. from ☉ by calculation at beginning,	989″,4
At the end,	1047, 9
The ☽'s viſible motion in latitude from the ☉,*	58, 5

In plate I. fig. I. let E☉C be a portion of the ecliptic § = 3352″,9 the moon's viſible motion from the ſun, during the eclipſe. Let ☽E be = the moon's viſible latitude from the ſun, at the beginning of the eclipſe, and CL the ſame at the end. Let ☽M be drawn parallel to E⊙C; then, ML = 58″.5 will be the moon's viſible motion in latitude from the ſun, during the eclipſe, and M☽L, the viſible angle of the moon with the ſun in the ecliptic.* Let ☽⊙ be the diſtance of the ⊙'s and ☽'s centers, at the beginning of the eclipſe, and ⊙L the ſame at the end; then the point at ⊙ will be the place of the ecliptic conjunction; and the line ⊙D, perpendicular to the ecliptic, and cutting the moon's viſible path ☽L in D, will be the moon's viſible latitude from the ſun, at the time of the ecliptic conjunction; and the line ⊙N, perpendicular to the viſible path, and cutting it in N, will be the leaſt diſtance of centers.

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* The moon's viſible motion as it ariſes from her horary motion and parallaxes in latitude, is not affected by the ſmall error in the latitude given by the tables.

§ See note, page 17.

☽M	3352″,9	3	5254206
: ML	58, 5	1	7675269
:: Radius		10	
: Tangent angle M☽L,	1° 0′ 2″	8	2421063
Sine angle M☽L,	1 0 2	8	2420401
: ML	58, 5	1	7675269
:: Radius		10	
: ☽L	3353, 4	3	5254868
☽L	3353″,4	3	5254868
: ☽⊙+L⊙	3904, 6	3	5915796
:: ☽⊙—L⊙	4, 0	0	6020600
: Difference ☽N and LN	4, 6	0	6681528

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* The true ſide ML is ſo ſhort in this ſmall figure, that it was neceſſary to increaſe it for the ſake of illuſtration; but every one knows that the calculations are not affected by it, the true length being uſed in them.

½☽L	1676″,7		
½ difference ☽N and LN	+— 2, 3		
☽N	1679, 0		
LN	1674, 4		
☽☉	1954, 3	3	2909912
: Radius		10	
:: ☽N	1679, 0	3	2250507
: Sine angle ☽☉N	59°13′ 9″	9	9340595
Angle N☽D = M☽L	+ 1 0 2		
Angle ☽☉D = ☽☉F	60 13 11		
	90		
Angle ☽☉E	29 46 49		
Radius		10	
: ☽☉	1954″,3	3	2909912
:: Sine angle ☽☉E	29°46′49″	9	6959990
: E☽	970, 5	2	9869902
☽'s viſible lat. from ☉ =	16′ 10, 5 S		
From ☽'s parallax in lat. from ☉	35 50, 1		
Sub. ☽'s viſi. lat. from ☉ now found	16 10, 5		
Remains ☽'s correct latitude,	19 39, 6 N		
☽'s lat. by Mayer's tables,	19 20, 7 N		
Error of the tables in ☽'s lat.	— 18, 9*		

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* As the time both of the beginning and end of the eclipſe is uſed in deducing this error, there is no need of finding CL, which would give the ſame.

Thus it appears, by the Greenwich obſervations of the beginning and end of this eclipſe, that the latitude of the moon, given by Mayer's tables, is 18″, 9 too ſmall,* which correction, being applied, will give her latitude at the beginning 19′ 39″, 6, and at the end 25′ 11″, 7. Hence the viſible difference of longitude, at the beginning, was 28′ 16″, 3, and at the end 27′ 36″, 7; and the true longitude by obſervation $3^{s.}$ 3° 6′ 33″, 3 and $3^{s.}$ 4° 12′ 22″, 4, the difference being 1° 5′ 49″, as by the tables. The true difference of longitude between the ſun and moon, at the beginning, was 151″, 3, the moon's longitude being the greateſt: Therefore, ☽'s horary motion from ☉ 2113″,3 : 1 hour = 3600″ :: 151″, 3 : $257\frac{3}{4}''$ = 4′ $17\frac{3}{4}''$; which being ſubtracted from $3^{h.}$ 40′ 11″ (the obſerved time of the beginning of the eclipſe at Greenwich) becauſe the moon had really paſſed the conjunction, leaves $3^{h.}$ 35′ $53\frac{1}{4}''$ for the apparent time of the true ecliptic conjunction.

Let us now aſſume $4^{h.}$ 44′ 4″ for the difference of meridians between Greenwich and Chelſea, and find the elements for Chelſea, at the end of the eclipſe, viz. At $11^{h.}$ 38′ 23″, anſwering to $4^{h.}$ 22 27 ′at Greenwich, according to this aſſumption.

At Chelſea at $11^{h.}$ 38′ 23″, A. M. apparent time.

The ſun's longitude,	$3^{s.}$	3°	5′	42 ,8
The moon's ecliptic longitude,	3	3	33	1, 8

The

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* By the Oxford obſervations of the beginning and end of this eclipſe, the error in latitude, in Mayer's tables, was — 20″, 6, differing but 0, 7 from the error found by the Greenwich obſervations.

The moon's lat. corrected by the Greenwich obſerva.			21	53,	2
The moon's horizontal parallax,			61	10,	0
The ſun's right aſcenſion,		93	22	25	
The right aſcenſion of the mid-heaven		87	58	10	
Angle *p*PZ,		177	58	10	
		= 2	1	50	
Complement		87	58	10	
The latitude of Chelſea,		42	25	11	
Reduced to the center,		42	10	20	
Complement = PZ		47	49	40	

Hence

The altitude of the nonageſimal degree,		71	17	6	
The longitude of the nonageſimal degree,	2	28	24	40	
Angle Z*p*☽ = B		5	8	21	
The moon's parallax in longitude from the ſun,			5	15,	8
The moon's parallax in latitude from the ſun,			19	32,	6
The ſum of ☉'s & ☽'s ſemi-diam.—4″,5 for infl. = ☉☽,				1959″,8	
The moon's viſible latitude from the ſun,				140,	6
The viſible diff. of long. of ☉'s and ☽'s centers = ☉E				1954,	7
				= 32′ 34″,7	
The moon's ecliptic long. by obſervation,	3ˢ·	3°	33	1,	7
The true diff. of long. of ☉ and ☽ according to the aſſumption,			27	18,	9
				= 1638″,9	

☽'s horary motion from ☉ 2113″, 3 : 1 hour = 3600″, :: 1638″, 9 : 2792″ = 46′ 32″. Subtract 46′ 32″ from 11$^{h.}$ 38′ 23″, the time of the end of the eclipſe at Chelſea, and the remainder 10$^{h.}$ 51′ 51″, A. M. will be the time when the true ecliptic conjunction happened. This ſubtracted from $3^{h.}$ $35'$ $53\frac{1}{4}''$, P. M. the time of the true ecliptic conjunction at Greenwich, leaves $4^{h.}$ $44'$ $2\frac{1}{4}''$ for the difference of meridians between theſe two places. To this add $26''$, the difference of meridians between Cambridge and Chelſea; the ſum, $4^{h.}$ $44'$ $28\frac{1}{4}''$ will be the difference between Cambridge and Greenwich, by the obſervations of this eclipſe.

Deductions from the obſervations of a tranſit of Mercury, November 5, 1743, for determining the difference of meridians between Paris* and Cambridge.

At Paris, November 5, 1743, a tranſit of Mercury was obſerved by

		1ſt. inter. contact. Apparent time.
Mr. Maraldi,	At the Royal Obſervatory,	$8^{h.}$ $40'$ $46''$
Mr. Caſſini, jun.		8 40 34
The Abbe de la Caille,	at College Mazarin,	8 40 38
Mr. Caſſini, ſen.	at Thury,	8 40 31
	The mean,	8 40 $37\frac{1}{4}$

	2d. inter. contact.	2d. exter. contact.
Mr. Maraldi,	$1^{h.}$ $10'$ $17''$	1 12 18
Mr. Caſſini, jun.	1 10 26	1 12 24
The Abbe de la Caille,	1 10 3	1 11 58
Mr. le Monnier, jun. near Port St. Honorè,	1 9 $54\frac{1}{2}$	1 12 2

Mr.

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* A bad ſtate of the atmoſphere prevented this tranſit from being obſerved at Greenwich.

Mr. Caſſini, ſen.	1$^{h.}$ 10′ 28″	*
Mean,	1 10 13$\frac{3}{5}$	1 12 10$\frac{1}{2}$

Hence it appears, that at the egreſs, Mercury was about 1′ 57″ in making his tranſit acroſs the ſun's limb, the half of which, viz. 58$\frac{1}{2}$″, added to the time of the ſecond internal contact, will give 1$^{h.}$ 11′ 12″, P. M. for the time of the central contact, at the egreſs; and, ſubtracted from the time of the firſt internal contact, will give the central contact at the ingreſs 8$^{h.}$ 39′ 38$\frac{3}{4}$″, A. M.

N. B. The place where Mr. Caſſini, ſen. obſerved is 6″ in time eaſt of the meridian of the Royal Obſervatory; the place where Mr. le Monnier, jun. obſerved, 2″ weſt. The times obſerved by them are here adjuſted to the meridian of the Obſervatory. The place where the Abbe de la Caille made his obſervations is ſo near to the meridian of the Royal Obſervatory, that the difference in time is imperceptible.

At 8$^{h.}$ 39′ 38$\frac{3}{4}$″, A. M. apparent time, at the Royal Obſervatory of Paris.

The ſun's longitude,	7$^{S.}$ 12° 32′ 57″,205
Mercury's geocentric ecliptic longitude,	7 12 45 5, 688
Mercury's geocentric lat. ſouth decreaſing,	10 48, 165
The ſun's right aſcenſion,	220 5 40
The right aſcenſion of the mid-heaven,	170 0 21
Angle *p*PZ,	99 59 39

= 80

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* Mr. Caſſini, ſen. was doubtful of the ſecond external contact; it is therefore omitted.

		= 80° 0′ 21″
Complement		9 59 39
Mercury's diſtance form the earth,	log.	4 829651
: The ſun's diſtance from the earth,	log.	4 995787
:: The ſun's horizontal parallax, 8″,58	log.	0 933487
: Mercury's horizontal parallax, 12, 578		1 099620

Mercury's horizontal parallax from the ſun,	3″,998
The ſun's ſemi-diameter,	16′ 11, 7
Mercury's ſemi-diameter,	5, 1
The ſun's horary motion,	2 30, 7
Mercury's geocen. horary mot. in the ecliptic retrog.	3 22, 12
Mercury's mot. upon ☉'s diſc with reſpect to the eclip.	5 52, 82
Mercury's horary motion in latitude geocentric,	51, 7
Latitude of the Royal Obſervatory at Paris,	48° 50 14
Reduced to the center,	48 35 26
Complement = PZ	41 24 34

Hence

The altitude of the nonageſimal degree,	50 2 38
The longitude of the nonageſimal degree,	5$^{S.}$ 28 11 44
Angle Zp☿ = B,	44 33 22
Mercury's parallax in longitude from the ſun,	2, 151
Mercury's parallax in latitude from the ſun,	2, 574
The ſun's ſemi-diameter = ☉☿	971, 7
Mercury's viſible lat. from the ſun's center = E☿	650, 739
The viſi. diff. of long. of ☉'s and ☿'s centers = ☉E,	721, 484
	= 12′ 37, 434
☉'s longitude,	7 12 32 57, 205

Add

Add viſi.diff.long.of ☉'s & ☿'s cent. ☿ being retro.	+ 12′ 1″,4
	$7^{s.}$ 12° 44 58, 605
Mercury's parallax in longitude from the ſun,	— 2, 151
Mercury's true long. by obſervation,	7 12 44 56, 454
The true diff.long.☉'s & ☿'s cen.by obſ.at $8^{h.}$ $39'38\frac{3}{4}''$,	11 59, 249
	60
	719, 249

N. B. The above references are to Plate I. Fig. I. and IV. ☽ being ſuppoſed changed for ☿. The various proceſſes are ſimilar to thoſe made in the foregoing ſolar eclipſes.

At $1^{h.}$ 11′ 12″ the time of the central contact of ☿ at the egreſs.

The ſun's longitude,	$7^{s.}$ 12° 44′ 19″,258
Mercury's geocentric ecliptic longitude,	7 12 29 50, 909
Mercury's geocentric latitude ſouth,	6 54, 174
The ſun's right aſcenſion,	220 16 58
The right aſcenſion of the mid-heaven,	238 4 58
Angle *p*PZ	31 55 2
Complement	58 4 58

Hence

The altitude of the nonageſimal degree,	24 16 35
The longitude of the nonageſimal degree,	7 1 43 37
Angle Z*p*☿ = B,	10 46 13
Mercury's parallax in longitude from the ſun,	0, 307

Mercury's

Mercury's parallax in latitude from the ſun,	3'',647
☉ ☿ as before,	971, 7
Mercury's viſible lat. from the ſun's center = E ☿	417, 821
The viſible diff. of long. of ☉'s and ☿'s centers = ☉E	877, 284
	= 14'37, 284
Mercury's true ecliptic long. by obſervation,	$7^{S.}$ 12°29 41, 667

Hence it appears, that Mercury's motion in geocentric longitude in the ecliptic, by obſervation, from his central contact at the ingreſs, to his central contact in the egreſs, was 15' 14'', 787. According to the horary motion in the elements, from whence the calculations were made, it was 15' 14'', 779. We may therefore take the motion given in the elements, as the true, and ſay, 5' 52'', 82 = 352'', 82 Mercury's horary motion upon the ſun's diſc with reſpect to the ecliptic : 1 hour = 3600'' :: 719'', 249 the true difference of longitude of ☉'s and ☿'s centers at the central contact at the ingreſs : 7339'' = $2^{h.}$ 2' 19''. This added to $8^{h.}$ 39' $38\frac{3}{4}$'', the time of Mercury's central contact at the ingreſs, will give $10^{h.}$ 41' $57\frac{3}{4}$'', A. M. apparent time, for the true ecliptic conjunction, by obſervation.

The ſame tranſit was obſerved by Profeſſor Winthorp,* at Cambridge, who ſaw the ſecond internal contact at $8^{h.}$ 17' 5'',

H A. M.

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* Profeſſor Winthrop, in a letter to Dr. Bliſs, Aſtronomer Royal, dated 20th June, 1763, and publiſhed in vol. LIX, of the Philoſophical Tranſactions, communicates his obſervations of this tranſit, with the expreſs deſign of ſettling the longitude of Cambridge, by comparing them with the European ones ; and he gives it as his opinion, that it might thereby be determined with more exactneſs, than by any obſervations that had been made uſe of, for that purpoſe.

He

A. M. apparent time, and the ſecond external contact at 8$^{h.}$ 18′ 58″; which makes the contact of Mercury's center, at the egreſs, at 8$^{h.}$ 18′ 1½″, A. M.

Suppoſe the difference of meridians between Paris and Cambridge to be 4$^{h.}$ 53′ 45″, then it was 1$^{h.}$ 11′ 46½″, P. M. at Paris, when it was 8$^{h.}$ 18′ 1½″, A. M. at Cambridge.

Elements at 1$^{h.}$ 11′ 46½″, P. M. at Paris, anſwering to 8$^{h.}$ 18′ 1½″, A. M. at Cambridge.

The ſun's longitude,	7$^{S.}$	12°	44′	20″, 701
Mercury's geocentric ecliptic longitude,	7	12	29	48, 968
Mercury's geocentric latitude ſouth,			6	53, 679
The ſun's right aſcenſion,		220	17	0
The right aſcenſion of the mid-heaven,		164	47	22
Angle pPZ,		105	12	38
		= 74	47	22
Complement		15	12	38
PZ, as before,		47	51	23

Hence

The altitude of the nonageſimal degree,		57	27	17
The longitude of the nonageſimal degree,	4	28	4	43
Angle Zp☿ = B,		74	25	5
Mercury's parallax in longitude from the ſun,				3, 246

Mercury's

He informs Dr. Bliſs, that his clock was adjuſted by correſpondent altitudes of the bright ſtar of Aries, taken the night before the tranſit, with a quadrant of two feet radius; and on the day of the tranſit, by correſpondent altitudes of the ſun; all which agreed within 5″; and that he allowed for the difference of the ſun's declination morning and afternoon.

Mercury's parallax in latitude from the ſun,	2″,154
☉ ☿,	971, 7
Mercury's viſi. lat. from the ſun's center = E ☿,	415, 833
The viſi. diff. of long. of ☉'s and ☿'s centers = ☉E,	878, 227
	= 14′ 38, 227
From the ſun's longitude,	$7^{s.}$ 12°44 20, 701
Subtract the viſi. diff. of long. of ☉'s & ☿'s centers,	14 38, 227
	7 12 29 42, 474
Subtract ☿'s parallax in longitude from ☉,	3, 246
Remains ☿'s geocent. eclip. long. by obſerv. accord. to the aſſumed diff. of meridians,	7 12 29 39, 228
The true diff. long. ☉'s & ☿'s cent. by obſerva. at $8^{h.}$ 18′ $1\frac{1}{2}$″, A. M.	14 41, 473
	60
	881, 473

352″, 82 : 3600″ :: 881″, 473 : 3994″ = $2^{h.}$ 29′ 54″.

Therefore,

From the time of ☿'s central contact by obſervation at Cambridge, at the egreſs,	$8^{h.}$ 18′ $1\frac{1}{2}$″, A.M.
Subtract	2 29 54
Rem. the time of the true ecliptic conj. by obſ.	5 48 $7\frac{1}{2}$, A. M.
The time of the true ecliptic conjunction at Paris by obſervation,	10 41 $57\frac{3}{4}$, A. M.
Difference = the difference of meridians between Paris and Cambridge,	4 53 $50\frac{1}{4}$

Sub. the diff. of merid. betw. Paris & Greenwich,	— 9′ 16″, A.M.
Remains the difference of meridians between Greenwich and Cambridge,	4h. 44 34$\frac{1}{4}$
The diff. by the obſervations of the ſolar eclipſe of Auguſt 5, 1766,	4 44 29$\frac{1}{2}$
Do. by the obſ. of the ſol. eclipſe of June 24, 1778,	4 44 28$\frac{1}{4}$
The mean	4 44 30$\frac{2}{3}$
Therefore, avoiding fractions, the difference of meridians may be called	4 44 31 ‡

I

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‡ In a letter to the preſent Aſtronomer Royal, publiſhed in the volume of the Philoſophical Tranſactions for 1781, I made the difference of meridians between Cambridge and Greenwich, by the ſolar eclipſe of Aug. 1766, to be 4h. 44′ 22″; and by the tranſit of Venus of June, 1769, to be 4h. 44′ 12″, comparing Mr. Hitchins's obſervation of the internal contact only, with Dr. Winthorp's. The mean of theſe two reſults, viz. 4h. 44′ 17″, I gave as the true difference. But as the ſun was low, when the obſervations of the tranſit were made at Greenwich, which cauſed an uncertainty attending them, not experienced where the altitude was conſiderable and the atmoſphere clear, I have ſuppoſed it not expedient to make uſe of the reſult from this phenomenon, in the foregoing paper.

In making my deductions from the ſolar eclipſe of 1766, in the above appendix, I have taken the mean between the time of the obſervations of the Aſtronomer Royal, and of his aſſiſtant; whereas, in the former calculations, I took thoſe only of the Aſtronomer Royal. I have alſo uſed 42° 23′ 28″, for the latitude of Cambridge, as determined by Profeſſor Williams, inſtead of 42° 25′; and with theſe alterations, having carefully gone over the work anew, I have found the difference of meridians, from this eclipſe, 4h. 44′ 29½″, inſtead of 4h. 44′ 22″.

The deductions from the tranſit of Mercury, and the ſolar eclipſe of 1778, were not made, when the above letter was written.

I ſuppoſed it proper to add this note, to ſhew the reaſon of the difference between the reſult in this paper, and that in the letter publiſhed in the Philoſophical Tranſactions.

I have been particular, in this appendix, in putting down the deductions from the obſervations which I have made uſe of, in ſettling the longitude of Cambridge from Greenwich, that any perſons, who may meet with this paper, and are able to go thro' this kind of calculation, may ſee what ſtreſs is to be laid upon the determinations. If the deductions are right, the near agreement between the three reſults is a ſtrong evidence of the goodneſs of the obſervations, and the dependence that may be made upon the mean reſult, for the difference of meridians between Greenwich and Cambridge; which, perhaps, is now aſcertained to as great exactneſs as the difference between almoſt any two places, where terreſtrial meaſurement has not been made uſe of.

J. W.

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